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OF THE
Pendulum Yarn Testing Machine

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THE DYNAMICAL THEORY OF THE PENDULUM YARN TESTING MACHINE

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During the course of certain work on yarn testing we had brought to our notice a form of autographic machine based on the ordinary pendulum machine. The records made by this machine had a peculiar shape, which we felt certain did not correspond to any peculiarity in the elastic properties of the yarn. This opinion was verified by testing a piece of thin copper wire ($\frac{5}{1000}$ in. diameter) in the machine. A similar curve was obtained.

The curves given by the machine were wavy, indicating quite wrongly an extraordinary relationship between stress and strain (even in the case of a copper wire!). In the case of ordinary worsted yarns the effect is so marked that no autographic arrangement is needed to show it: it can be seen by simply watching during a test the pointer of an ordinary Baër, Schopper or Goodbrand machine. The pointer moves from rest, suddenly stops for an instant, moves on again, stops again, and so on: the motion appearing to consist of a series of jerks. These jerks often become less marked after a time, the motion in the later stages appearing much smoother.

This peculiar motion is familiar to the more observant users of the machines. The writers have heard the opinion expressed that it is due to irregular stretching of the yarn due in some way to the irregular slipping of fibre over fibre. This view is, however, erroneous. The effect is due entirely to the inertia of the moving parts. In the present paper a theory of the motion will be worked out. This theory will give a complete explanation of the observed effect, and will lead to results of interest to the users and designers of testing machines.

We will consider the ideal case of a yarn of perfect elastic properties tested in a machine of the pendulum type in the usual manner—by causing the lower yarn clamp to descend at a uniform speed. The arrangement is shown diagrammatically in Fig. 1. If the uniform speed of descent of the lower yarn clamp denoted by v , then after a time t the lower end of the yarn will have descended a distance vt . Suppose that in this time the pendulum has moved from its equilibrium position through an angle θ . If a denote the radius of the pulley over which runs the flexible chain, cord, or strip carrying the upper yarn clamp, then the distance descended by the upper end of the yarn is $a\theta$. The extension of the yarn is therefore $vt - a\theta$. If E denotes the force per unit extension of the yarn, and if the initial tension is zero, the tension F after a time t is given by the equation

$$F = E(vt - a\theta) \dots \dots \dots (1)$$

The assumption usually made is that the indicated tension is equal to the actual tension of the yarn. Before proceeding to a more accurate theory we will work out the consequences of this assumption.

As we have seen, this theory does not fit the observed facts. We will, therefore, develop a theory which takes account of the inertia of the pendulum. At any instant we have the moment of the tension of the yarn about the axis, of magnitude Fa , acting in a clockwise direction (see Fig. 1), and the moment of the gravitational forces of magnitude $Mgh \sin \theta$ acting in an anti-clockwise direction. These two quantities, instead of being equal as stated in equation (2), will differ by an amount equal to the rate of increase of the angular momentum of the pendulum. If k denote the radius of gyration of the pendulum about the axis, we have, therefore,

$$Mk^2 \ddot{\theta} = Fa - Mgh \sin \theta \quad (6)$$

Inserting in this equation the value of F given by equation (1), we obtain the equation

$$Mk^2 \ddot{\theta} = Eavt - Ea^2 \theta - Mgh \sin \theta \quad (7)$$

This equation is not integrable in finite terms. The solution by means of an infinite series, though simple in principle, is extremely laborious in practice. It will suffice, however, for our purpose to solve the equation for the special case where θ is so small that it does not differ appreciably from $\sin \theta$. In this case we may write

$$Mk^2 \ddot{\theta} = Eavt - (Ea^2 + Mgh) \theta \quad (8)$$

This is most simply solved by differentiating once, and taking temporarily the new variable

$$\eta = \dot{\theta} - \frac{Eav}{Ea^2 + Mgh} \quad (9)$$

The equation thus reduces to

$$\ddot{\eta} = - \left(\frac{Ea^2 + Mgh}{Mk^2} \right) \eta \quad (10)$$

The integral of this is

$$\eta = A \cos \frac{2\pi t}{T} \quad (11)$$

where A is an arbitrary constant and T is given by the equation

$$T = 2\pi \left(\frac{Mk^2}{Ea^2 + Mgh} \right)^{\frac{1}{2}} \quad (12)$$

Since the pendulum starts from rest $\dot{\theta} = 0$ when $t = 0$, so that we must have

$$A = - \frac{Eav}{Ea^2 + Mgh} \quad (13)$$

and, introducing the quantity Ω defined by equation (5)

$$\dot{\theta} - \Omega = - \Omega \cos \frac{2\pi t}{T} \quad (14)$$

We thus see that the angular velocity of the pendulum, instead of having the steady value Ω as indicated by the previous theory, oscillates about this value, ranging from zero at $t = 0, T, 2T \dots$ to 2Ω at $t = \frac{1}{2}T, \frac{3}{2}T \dots$. The new theory leads to a result in agreement with the observed facts—equation (14) indicates a series of periodic momentary stoppages of the pendulum such as are actually observed.

Integrating equation (14), we obtain the complete solution of (8). Assuming $t = 0$, when $\theta = 0$, we obtain

$$\theta = \Omega t - \frac{\Omega T}{2\pi} \sin \frac{2\pi t}{T} \quad (15)$$

The relation between this result and the result obtained on the previous theory is very simple. Equation (15) consists merely of equation (4) with the addition of a periodic term. The effect of the inertia of the pendulum is to make its position alternately behind and in front of the position indicated by the theory which neglects the effect of inertia.

The period T of these fluctuations of velocity is given by equation (12). This period may readily be proved to be equal to the natural period of oscillation of the pendulum under the joint action of gravity and the elastic forces of the yarn, *i.e.*, the period of the oscillations which would be set up if the lower end of the yarn were displaced rapidly downwards through a short distance and held in its new position.

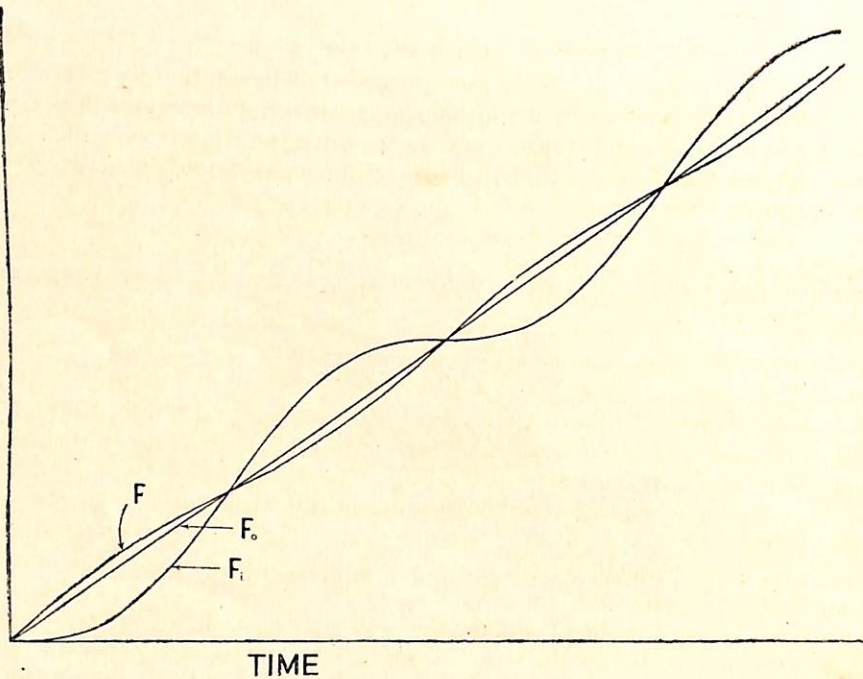


FIG. 2.

The period of the free oscillations of the pendulum is given by

$$T_o = 2\pi \left(\frac{k^2}{gh} \right)^{\frac{1}{2}}$$

so that

$$T = T_o \left(\frac{Mgh}{Ea^2 + Mgh} \right)^{\frac{1}{2}} \quad \dots \quad (16)$$

The tension of the yarn is readily obtained from equations (1) and (15). We get at once

$$F = Et(v - a\Omega) + \frac{Ea\Omega T}{2\pi} \sin \frac{2\pi t}{T}$$

which reduces by virtue of equation (5) to

$$F = \frac{Mgh\Omega t}{a} + \frac{Ea\Omega T}{2\pi} \sin \frac{2\pi t}{T} \quad \dots \quad (17)$$

This is the true tension of the yarn. It will not in general be equal to

the tension indicated by the machine. Denoting this indicated tension by F_i , we have

$$F_i a = Mgh\theta \quad \dots \dots \dots (18)$$

so that from (15) we readily deduce the expression

$$F_i = \frac{Mgh\Omega t}{a} - \frac{Mgh\Omega T}{2\pi a} \sin \frac{2\pi t}{T} \quad \dots \dots \dots (19)$$

There is a third value of the tension; like F_i , a fictitious value. It is the value calculated on the theory that the inertia is negligible. Denoting this by F_o , we have, from (2), (4) and (5)

$$F_o = \frac{Mgh\Omega t}{a} \quad \dots \dots \dots (20)$$

If F , F_i and F_o be plotted against the time, we get three graphs of the kind shown in Fig. 2. Near the commencement of the motion F_i is less than F_o owing to the slowness with which the pendulum gathers up speed. On the other hand, F , the true tension, is greater than F_o owing to the fact that the lagging of the pendulum causes the rate of extension of the yarn to be greater than indicated by the simple "non-inertial" theory. As the pendulum gains speed the rate of extension of the yarn decreases, as does therefore the rate of increase of F . At the end of a quarter period F , F_o and F_i are all increasing at the same rate ($Mgh\Omega/a$), though F is still greater than F_o , so that the pendulum still gains speed and F still increases at a decreasing rate. At the end of a half period F , F_o and F_i become equal. Beyond this point the pendulum is retarded, the rate of extension of the yarn, and therefore the rate of increase of F begin to increase, till at the end of a three-quarter period F , F_o and F_i are all increasing at the same rate ($Mgh\Omega/a$), F being now less than F_o . At the end of a complete period the angular velocity of the pendulum and the rate of increase of F_i become zero, and F , F_o and F_i become equal again. The motion continues in a similar manner, though as the angle of displacement becomes larger the quantitative accuracy of the above theory becomes less. A qualitative agreement between theory and practice still continues, however, and there is no difficulty in observing the momentary stoppages indicated by the theory, when the angle of displacement is comparatively large (e.g., 30°). The oscillations do, however, tend to damp out in the later stages of the motion, owing partly to friction and partly to the imperfect elasticity of the yarn.

From a practical point of view it is important to be able to estimate the magnitude of the error due to inertia. From (5), (17) and (19) we have

$$F - F_i = \frac{EvT}{2\pi} \sin \frac{2\pi t}{T} \quad \dots \dots \dots (21)$$

The maximum value of the error is, therefore, $EvT/2\pi$. When $t = \pi/2, 5\pi/2, \dots$ the true tension is greater than the indicated tension, and when $t = 3\pi/2, 7\pi/2, \dots$ the true tension is less than the indicated tension by this amount. The quantity EvT may be interpreted quite simply. It is equal to the increase of tension of the yarn due to an extension equal to the distance descended by the lower end of the yarn in one complete period.

In many machines the diameter of the lever roller is such that for ordinary lengths of woollen and worsted yarns the extension of the yarn is

much greater than the descent of the upper yarn clamp. It may readily be shown that

$$\frac{\text{descent of upper end of yarn}}{\text{extension of yarn}} = \frac{Ea^2}{Mgh} \quad \dots \quad (22)$$

so that in this case the period of the fluctuations will not differ appreciably from the free period of the pendulum. Since, moreover, the descent of the lower clamp is not very different from the extension of the yarn, we have the following rule: *The maximum error due to inertia is slightly greater than the fraction $\frac{1}{2}\pi$ (16%, or roughly one-sixth) of the reading indicated at the first momentary stopping place.* In the case of a machine where the lever roller is large the error may be considerably greater than this amount. It does not follow, however, that a large lever roller tends to produce a large inertia error. Equation (21) may be written

$$F - F_i = \frac{EvT_o}{2\pi} \left(\frac{Mgh}{Ea^2 + Mgh} \right)^{\frac{1}{2}} \sin \frac{2\pi t}{T} \quad \dots \quad (23)$$

From this we see that for a fixed value of the free period the error is less the larger the lever roller. In the case of an actual yarn, a large lever roller will throw the onus of maintaining the fluctuations very largely on the imperfect elastic forces of the yarn, and so tend further to diminish the inertia effect. We may point out, however, that a large lever roller has the disadvantage that the rate of extension of the yarn depends very largely on the extensibility of the yarn.

In plotting the graph of F in Fig. 2 we have assumed that $Ea^2 < Mgh$ (i.e., that the lever roller is small), so that the fluctuations of F are smaller than those of F_i and dF/dt never vanishes. With a large lever roller dF/dt might periodically become negative. With an actual yarn this is unlikely to happen because of damping produced by imperfect elasticity.

It follows from the above that in strength tests the speed should always be sufficiently small for the reading at the first momentary stopping place to be small compared with the mean strength. In a large number of tests the positive and negative errors tend to cancel. Moreover, in the neighbourhood of the breaking point, the elasticity is generally (though not always) very imperfect, so that the oscillations will damp out.

The inertia effect cannot, therefore, be regarded as seriously affecting the accuracy of the pendulum machine as ordinarily used for strength testing. It absolutely rules out the machine, however, as the basis of an autographic machine, where a complete relation between stress and strain is required and where there is no smoothing out of errors by averaging.

It is worthy of note that the magnitude of the speed of working does not affect the amplitude of the relative variations of the angular velocity of the pendulum. However slowly the machine is worked the angular velocity fluctuates between zero and double its mean value. This is well shown by some curves obtained quite recently by Hemmerling (*Textilberichte*, April, 1923, p. 166), by means of an adaptation of the Schopper machine. The speed of working has in all cases been sufficiently small to prevent any very serious divergence from the mean line, but the temporary halts are clearly marked, even in the case where the speed has been so small that the divergence from the mean line is absolutely negligible.

It is natural to enquire how the inertia error may be reduced by altering the design of the instrument. We have seen that the error is proportional

to the time of oscillation of the pendulum, which, therefore, must be made as small as possible. Any alteration made must be such as to leave unaltered the relation between the indicated force and the angle of deflection, *i.e.*, the range of the instrument. Consider a particle of the pendulum of mass m at a distance r from the axis. Let us suppose this to be moved in a plane passing through the axis till its distance is fr , where f is a proper fraction. At the same time let its mass be increased to m/f . The range of the instrument will not be affected. The moment of inertia will, however, be diminished by an amount $\frac{1}{2}mr^2(1-f)$. The alteration will, therefore, result in a diminution of the time of oscillation. If we suppose every particle to be changed in this way we get a new pendulum whose period instead of being T is only $T\sqrt{f}$ and whose mass instead of being M is M/f . The new pendulum will be a different shape. Its projection on a plane at right angles to the axis will be unaltered in shape, all parts being reduced in size in the ratio $1:f$. The increased amount of material will have to be accommodated by expanding the linear dimensions in directions parallel to the axis. For exact adjustment this expansion would have to be in the ratio $f^4:1$, though this might not be absolutely necessary. Thus if one part of the pendulum consisted of the usual type of cylindrical weight (with its axis parallel to the axis of rotation) and we wished to halve the period, the weight would have to be made four times as heavy, and the distance of its centre of gravity from the axis would have to be reduced to one-quarter. For exact adjustment the diameter of the weight would have to be halved and its length increased sixteenfold. A change of shape of this kind would only be necessary, however, when the distance from the axis became so small that the moment of inertia of the weight about its own axis became comparable with its moment of inertia about the axis of rotation.

So far as simple dynamical theory indicates, the period may be made as small as we like by making the pendulum heavy, of small dimension in directions perpendicular to the axis, and of large dimension in directions parallel to the axis. In practice we are restricted by the large increase of mass necessary (involving increased friction). Thus to halve the period a fourfold increase of mass is necessary—to make the period a third, a ninefold increase of mass is necessary, and so on. It may be regarded as scarcely practicable to make the time of oscillation much less than one second.

